

Two-stream Instability in Pulsar Magnetospheres

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Abstract. Creation of electron-positron pairs near the pulsar surface and the parameters of plasma in pulsar magnetospheres are discussed. It is argued that the pair creation process is nonstationary, and the pair plasma that flows out from the pulsar environment is strongly non-homogeneous and gathers into separate clouds. Plasma instabilities in the outflowing plasma are reviewed. The two-stream instability that develops due to strong nonhomogeneity of the outflowing plasma is the most plausible reason for the generation of coherent radio emission of pulsars. The development of the two-stream instability in pulsar magnetospheres is considered.

1. Introduction

Current models for the generation of coherent radio emission of pulsars require the development of plasma instabilities in the pulsar magnetospheres, the most studied of which is the two-stream instability (for a review, see Arons 1981a,b; Melrose 1981, 1993, 1995; Usov 1981; Asseo 1996). The two-stream instability in the pulsar magnetospheres has been considered in many papers (e.g., Ruderman & Sutherland 1975; Benford & Buschauer 1977; Buschauer & Benford 1977; Cheng & Ruderman 1977; Arons 1981; Usov 1987; Ursov & Usov 1988; Asseo 1993; Asseo & Melikidze 1998; Melikidze, Gil & Pataraya 2000).

The two-stream instability can arise when two plasma flows travel through each other:

$$\begin{array}{c} \leftarrow \mathbf{v}_1, n_1 \\ \mathbf{v}_2, n_2 \rightarrow \end{array}$$

If the density of one flow is much higher than the density of the other (for example, $n_1 \gg n_2$), the low-density flow (component 2) calls the beam while the high-density flow (component 1) calls the plasma. In the magnetospheres of pulsars, both the plasma-beam interaction and the interaction of two plasmas with more or less equal densities may occur (see below). The two-stream instability may lead to formation of plasma bunches that generate the radio emission via curvature mechanism (Ruderman

& Sutherland 1975; Cheng & Ruderman 1980; Ochelkov & Usov 1984). Besides, the radio emission of pulsars may be generated directly in the process of the instability development (Asseo, Pellat & Rosado 1980; Asseo, Pellat & Sol 1983). Before to discuss the two-stream instability in the pulsar plasma and the generation of coherent radio emission of pulsars, we review both the physical processes that are responsible for the filling of pulsar magnetospheres with plasma and the plasma parameters.

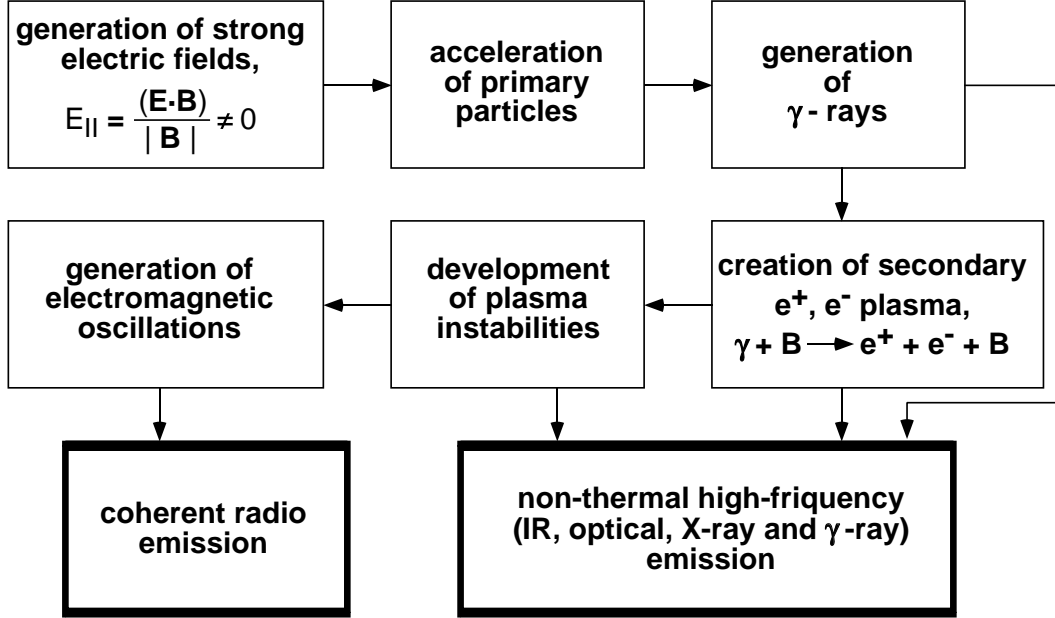
2. Physical processes and parameters of plasma in pulsar magnetospheres

A common point of all acceptable models of pulsars is that the spin-down power of strongly magnetized neutron stars is the energy source of non-thermal emission of pulsars. The rotational energy of the neutron star is transformed into the pulsar emission by a long sequence of processes.

2.1. Physical processes in pulsar magnetospheres

Strong electric fields are generated in the magnetospheres of rotating magnetized neutron stars (for a review, see Michel 1991). The component of the electric field $\mathbf{E}_{\parallel} = (\mathbf{E} \cdot \mathbf{B})\mathbf{B}/|\mathbf{B}|^2$ along the magnetic field \mathbf{B} is non-zero. Primary particles are accelerated by this electric field to ultrarelativistic energies and generate γ -rays. Some of these γ -rays are absorbed by creating secondary e^+e^- pairs. The created pairs screen the electric field \mathbf{E}_{\parallel} in the pulsar magnetosphere everywhere except for compact regions. The compact regions where \mathbf{E}_{\parallel} is unscreened are called gaps. These gaps are "engines" that are responsible for the non-thermal radiation of pulsars. The gaps are located either near the magnetic poles of pulsars (polar gaps) or near their light cylinders (outer gaps). Outer gaps may act as a generator of non-thermal radiation of pulsars only if the period of the pulsar rotation is small enough, $P < P_{\text{cr}} \simeq \text{a few} \times 0.1 \text{ s}$. For typical pulsars with $P > P_{\text{cr}}$, the polar gap model has no an alternative. In the polar gap model the sequence of processes that leads to the generation of non-thermal emission of pulsars is the following:

Physical processes in pulsar magnetospheres



The density of primary particles accelerated in the polar gap is about the Goldreich-Julian density $n_{\text{GJ}} = |\mathbf{\Omega} \cdot \mathbf{B}| / 2\pi c e$, where $\mathbf{\Omega}$ is the angular velocity of the pulsar rotation. The value of n_{GJ} is determined so that the electric field E_{\parallel} in the outflowing plasma is screened completely if the charge density is equal to en_{GJ} . Therefore, the accelerating field E_{\parallel} arises from deviations from this density. Such a deviation may be because of (1) the inertia of particles (Michel 1974), (2) the curvature of the magnetic field lines (Arons 1981), the General Relativity effects (Muslimov & Tsygan 1992, Beskin 1992), and the binding of particles within the solid surface of a strongly magnetized neutron star (Ruderman & Sutherland 1975; Cheng & Ruderman 1980; Usov & Melrose 1995, 1996). When both the neutron star surface is cold enough and the surface magnetic field is strong enough, there is no emission of particles from the neutron star surface, and the binding of particles is responsible for the E_{\parallel} field in the polar gaps (e.g., Usov & Melrose 1996). In the case when particles flow freely from the stellar surface, the field E_{\parallel} is mainly due to the General Relativity effects (i.g., Harding & Muslimov 1998). The second case is applicable to typical pulsars while the former case may be applied to pulsars with very strong magnetic fields at their surface, $B_s \sim 10^{13}$ G or more.

Ultrarelativistic primary particles in the process of their outflow move practically along the magnetic field lines and generate γ -rays via both curvature radiation and magnetic Compton scattering. These γ -rays are absorbed in strong magnetic fields of pulsar magnetospheres and create e^+e^- pairs. Created pairs very quickly lose the momentum component transverse to the magnetic field

due to synchrotron losses, and their distribution becomes one-dimensional, i.e., $\mathbf{v} \parallel \mathbf{B}$. Hence, to develop the theory of coherent radio emission of pulsars it is necessary to investigate both the parameters of ultrarelativistic one-dimensional plasma outflowing from the polar cap regions and instabilities of this plasma.

2.2. Parameters of pulsar plasma

Electrons and positrons that flow away from the neutron star vicinity with relativistic speeds may be divided into two components: a secondary e^+e^- plasma and an extremely high-energy beam of primary particles (see Fig. 1). Particles of the beam may be either electrons or positrons, depending on the direction of the electric field \mathbf{E}_{\parallel} in the polar gap. For the e^+e^- plasma (denoted by p) and the ultrarelativistic beam (denoted by b) the typical values of the Lorentz factor Γ and the density n are, respectively, (Ruderman & Sutherland 1975; Arons 1981a,b, 1984; Machabeli & Usov 1989; Michel 1991)

$$\Gamma_p \simeq (2 - 3)R_c/R \simeq \text{a few} \times (1 - 10^2), \quad (1)$$

$$\Gamma_b \simeq e\Delta\varphi/m_e c^2 \simeq \text{a few} \times 10^6, \quad (2)$$

$$n_p \simeq (\Gamma_b/2\Gamma_p)n_b, \quad (3)$$

$$n_b \simeq n_{\text{GJ}} = \Omega B / (2\pi c e) \simeq 10^{11} (R/r)^3 \text{ cm}^{-3}, \quad (4)$$

We have adopted here the following characteristic values for the pulsar parameters: $\Omega \simeq 10 \text{ s}^{-1}$; $B \simeq 10^{12} (R/r)^3$ G is the magnetic field in the magnetosphere; R_c is the curvature radius of the magnetic field lines in the polar cap; $R \simeq 10^6$ cm is the neutron star radius; and $\Delta\varphi$ is the potential drop across the polar gap.

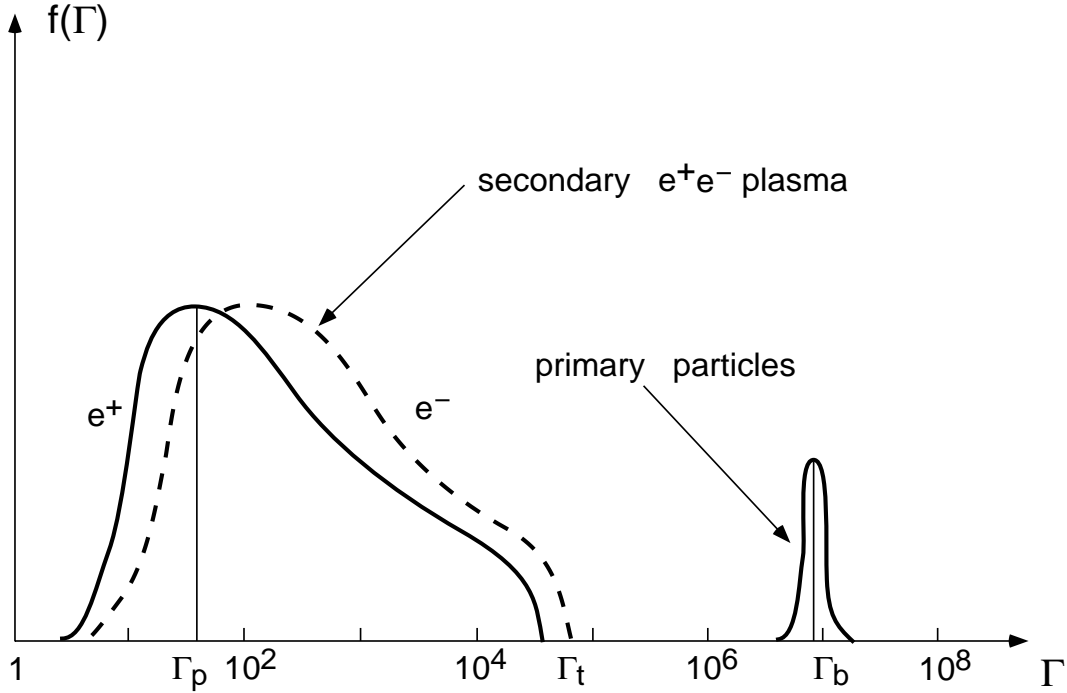


Fig. 1. The energy spectrum of the basic components of the plasma particles ejected from the polar gap region into the pulsar magnetosphere. The spectra of secondary electrons (dashed line) and secondary positrons (solid line) are shifted relatively each other.

2.3. Nonstationarity of the plasma outflow

It was suggested long time ago that the process of pair creation near the pulsar surface is strongly nonstationary (Sturrock 1971; Alber et al. 1975; Ruderman and Sutherland 1975). Such a nonstationarity was discussed in detail in the outstanding paper by Ruderman and Sutherland (1975) where the first self-consistent model of pulsar radiation was presented. In the Ruderman-Sutherland model, the creation of pairs occurs in narrow spark discharges, separated from one another by a distance of the order of the polar gap height H (characteristic value of H is $\sim 10^4$ cm). The secondary e^+e^- plasma, produced by absorption of γ -rays in a spark discharge, gathers into an outflowing jet that has the discharge at its base. Such a discharge is strongly nonstationary on a time scale of $\sim 30(H/c)$ (Ruderman & Sutherland 1975; Cheng & Ruderman 1980). The discharge nonstationarity results in a strong modulation of the outflowing pair plasma along the jet. In fact, these jets consist of plasma clouds spaced by $L_1 \simeq 30H \simeq 0.3R$.

However, it is not only the nonstationarity of discharges that can lead to inhomogeneity of the outflowing plasma along the fixed magnetic field lines. Another reason for such an inhomogeneity is the displacement of discharges caused by the curvature of the magnetic field lines (Ruderman & Sutherland 1975). The displacement is directed toward the magnetic axis of the pulsar. The speed with which a spark discharge moves across the polar cap is estimated (Ruderman & Sutherland 1975; Cheng & Rud-

erman 1980) as $v_d \simeq cH/2R_c$. As already noted above, the average distance between two discharges is $\sim H$. During the time $\tau_d \simeq H/v_d \simeq 2R_c/c$, a discharge is displaced by the distance of $\sim H$ and takes a position close to the initial position that of a neighboring discharge, which during the same time is also displaced by $\sim H$ toward the pulsar magnetic axis. As a result of these displacements of discharges, the plasma outflowing along any fixed magnetic field lines becomes modulated with a characteristic length $L_2 \simeq c\tau_d \simeq 2R_c$. Since, according to Ruderman and Sutherland (1975) and Beskin (1982), in the polar gap the discharges are separated by intervals within which the process of pair creation is suppressed, the plasma density in between the clouds falls down almost to zero. Therefore, the outflowing plasma gathers into separate clouds spaced by $L_2 \simeq 2R_c$ along the direction of the plasma outflow. In turn, these large clouds consist of small clouds with the lengths and the spaces between them about $L_1 \simeq 0.3R$.

The idea that the process of pair creation near the polar caps of pulsars is nonstationary has got recently the strong support (e.g., Muslimov & Harding 1997; Harding & Muslimov 1998). In these papers, the model of the polar gaps of typical pulsars which is rather close to reality is developed. In this model, it was shown that both stable acceleration of primary particles and stable creation of pairs are not possible. The characteristic length of the outflowing plasma modulation is expected more or less comparable with L_1 .

3. Possible instabilities in pulsar plasma

Development of instabilities in a magnetized plasma when transverse electromagnetic waves (t -waves) are generated depends on the ratio ω_p/ω_B , where

$$\omega_p = \sqrt{\frac{4\pi e^2 n_p}{m_e}} \simeq 5.64 \times 10^4 n_p^{1/2} \text{ s}^{-1}, \quad (5)$$

$$\omega_B = \frac{eB}{m_e c} \simeq 1.76 \times 10^7 B \text{ s}^{-1}, \quad (6)$$

are the Langmuir frequency of the plasma and the cyclotron frequency, respectively. At the pulsar surface for the typical parameters of both the pulsar plasma and the strength of the magnetic field ($n_{p0} \sim 10^{14} \text{ cm}^{-3}$ and $B_{s0} \sim 10^{12} \text{ G}$), from equations (5) and (6) we have $\omega_{p0}/\omega_{B0} \sim 10^{-7}$.

Since the difference between the phase velocity (v_{ph}) of t -waves and the speed of light is extremely small at the pulsar surface (e.g., Lominadze, Machabeli & Usov 1983),

$$c - v_{ph} \sim c(\omega_{p0}/\omega_{B0})^2 \Gamma_p \sim 10^{-13} c \quad (7)$$

in the frame of the pair plasma, particles of both the plasma and the beam cannot be in a resonance with t -waves that propagate in the pulsar plasma, and resonance instabilities (for example, the cyclotron instability) do not develop in the neutron star vicinities.

Besides, the energy density of plasma ($\varepsilon_p \simeq m_e c^2 n_p \Gamma_p$) is very small in comparison with the energy density of the magnetic field ($\varepsilon_B = B^2/8\pi$),

$$\varepsilon_p/\varepsilon_B \sim 2(\omega_{p0}/\omega_{B0})^2 \Gamma_p \sim 10^{-13} \quad (8)$$

Therefore, hydrodynamic instabilities are also suppressed near the pulsar surface by the strong magnetic field.

In the near zone of the pulsar magnetosphere ($r < c/\Omega$), we have $n_p \propto B \propto r^{-3}$. The ratio ω_p/ω_B increases with increase of the distance from the pulsar, $\omega_p/\omega_B \propto r^{3/2}$. Near the light cylinders of pulsars the ratio ω_p/ω_B is of the order of unity, and the outflowing plasma may be unstable with respect to excitation of t -waves (e.g., Machabeli & Usov 1979, 1989; Lominadze et al. 1983; Lyutikov, Machabeli & Blandford 1999 and references therein). At the distance from the pulsar in the range $R \leq r \ll c/\Omega$ only the two-stream instability may be developed.

Observations of pulsars suggest that the radio emission regions are far inside the light cylinder (e.g., Gil & Kijak 1993; Kramer et al. 1994; Kijak & Gil 1997),

$$r_{\text{radio}} \simeq 50R \simeq (0.02 - 0.04) c/\Omega \ll c/\Omega, \quad (9)$$

Therefore, the development of the two-stream instability in the magnetospheres of pulsars is the most plausible reason of their extremely intense radio emission.

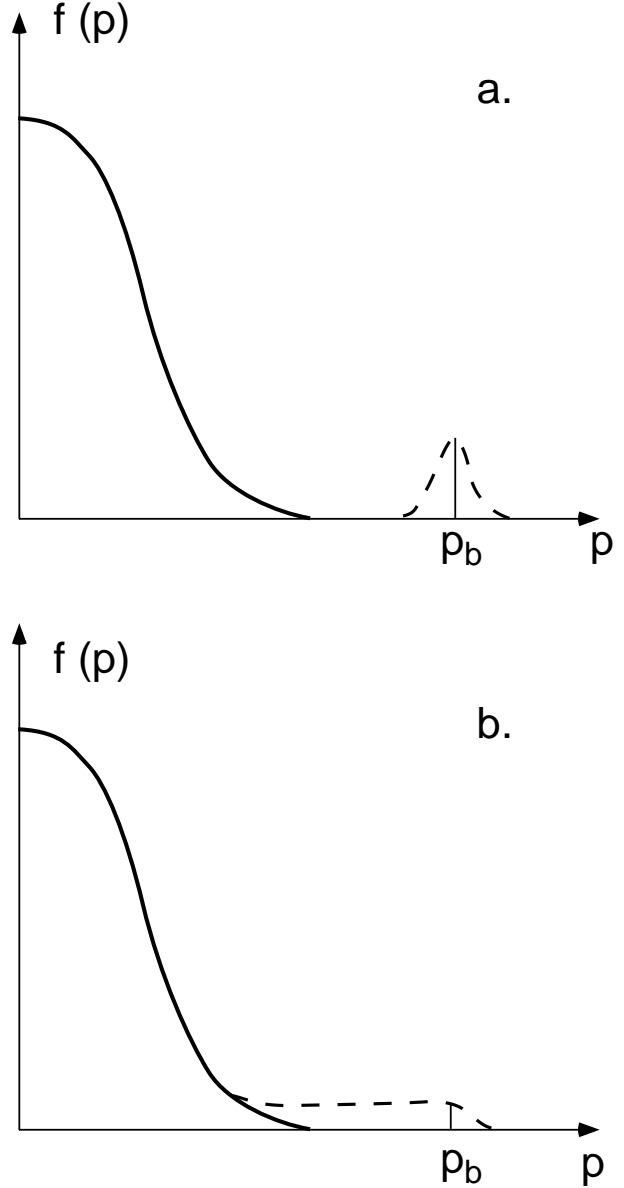


Fig. 2. The momentum distribution function of particles for a plasma-beam system: (a) prior and (b) after the development of the two-stream instability.

4. Two-stream instability

The two-stream instability may occur only if (1) there is a "hump" on the momentum distribution function $f(p)$, i.e. a region having $\partial f/\partial p > 0$, and (2) in the frame where plasma is at rest the mean momentum of the "hump" particles (p_b) is large enough in comparison with the mean momentum of the plasma particles (the Penrose criteria). Particles, which are in the inside slope of the "hump" where $\partial f/\partial p > 0$, produce longitudinal (l) waves with the frequency $\omega = (\mathbf{k}, \mathbf{v})$ (the Cherenkov resonance), where \mathbf{k} is the wave vector (see Fig. 1). These particles thus lose their energy. As a result the inside slope of the "hump" begins to move into the region of even lower velocities, and

the distribution function of the "hump" particles becomes "plateau-shaped" (see, Fig. 1). The further generation of l -waves is cut off. The plateau formation and the cut-off in the plasma wave generation is called the quasi-linear relaxation. The energy density of l -waves generated in the process of quasi-linear relaxation is of the order of the energy density of the "hump" particles.

A strong magnetic field has no an influence on the development of the two-stream instability in the one-dimensional plasma outflowing from the polar cap regions of pulsars, and therefore, this instability may occur everywhere in the pulsar magnetosphere if the mentioned above conditions for its development are realized. Since there are several components of particles in the pulsar plasma that move with different speeds, the idea that the two-stream instability is responsible for the coherent radio emission of pulsars is very natural.

The two-stream instability could be developed due to the following three reasons. The first one is the interaction between the beam of primary particles and the secondary e^+e^- plasma (Ruderman & Sutherland 1975). The second one is the interaction between electrons and positrons of the secondary plasma itself (Cheng & Ruderman 1977). At last, the two-stream instability may be triggered in the inhomogeneous pulsar plasma when the outflowing plasma clouds disperse and overlap each other (Usov 1987; Ursov & Usov 1988; Asseo & Melikidze 1998).

4.1. The interaction between the primary beam and the secondary plasma

The ultrarelativistic beam of primary particles has the distribution function that obeys the conditions that are necessary for the two-stream instability development (see Fig. 1). Therefore, the generation of radio emission of pulsars was initially associated with the ultrarelativistic beam of primary particles (Ruderman & Sutherland 1975). However, further more precise estimates of the instability growth rate have shown that it is impossible. Indeed, the characteristic time for development of the two-stream instability in the pulsar frame is (Benford & Buschauer 1977; Egorenkov, Lominadze & Mamradze 1983)

$$\tau_i \simeq (n_p/n_b)^{1/3} \Gamma_b \Gamma_p^{1/2} \omega_p^{-1}. \quad (10)$$

From equations (1)-(4), (5) and (10), for typical pulsars the value of τ_i is about $10^{-4}(r/R)^{3/2}$ s at the distance r from the neutron star. For the instability development, it is necessary that τ_i is, at least, a few times less than $\tau_0 = r/c \simeq 3 \times 10^{-5}(r/R)$ s, the characteristic time of the plasma outflow from the distance r . However, the reverse relation between τ_i and τ_0 holds. Hence, the two-stream instability that, in principal, could be caused by the primary beam because its distribution function forms a "hump" with $\partial f/\partial p > 0$, in fact, has not enough time to be developed in the pulsar magnetospheres. This is because both

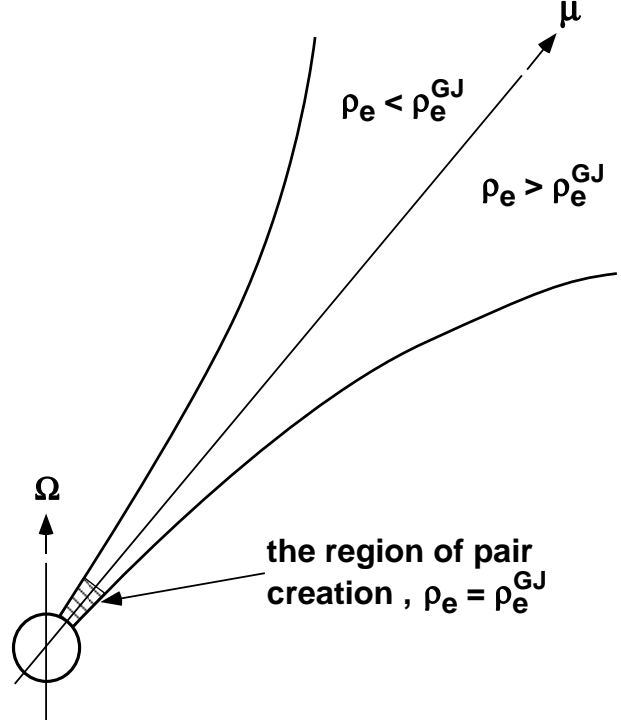


Fig. 3. Charge flow separation of the pulsar plasma moving outward along the curved magnetic field lines; μ is the magnetic dipole moment.

the beam density is too low and the Lorentz-factor of the beam is too high.

4.2. The interaction between the electrons and the positrons of the secondary plasma

The secondary e^+e^- plasma produced by photons initially has identical distribution functions in its components (electrons and positrons). In this case, the pair plasma is neutral, and the charge density ρ_e is determined by the primary beam and equals to $\rho_e^{\text{GJ}} = en_{\text{GJ}}$. In the process of the plasma outflow the charge density ρ_e is $\propto n \propto B$ while ρ_e^{GJ} is $\propto B \cos \chi$, where χ is the angle between Ω and \mathbf{B} . The magnetic field lines are curved, and therefore, the angle χ varies with the distance from the pulsar. This leads to a deviation of ρ_e from ρ_e^{GJ} and generation of the electric field along the magnetic field (see Fig. 3). This field accelerates one of the components of the secondary plasma and decelerates the other, and the distribution functions of these components shift to each other. The difference between the mean velocities of the electrons and the positrons is (Cheng & Ruderman 1977)

$$|v_+ - v_-| \simeq \frac{n_b}{n_p} \left[\frac{\Omega \cdot \mathbf{B}}{(\Omega \cdot \mathbf{B})_0} - 1 \right] c, \quad (11)$$

where $(\Omega \cdot \mathbf{B})_0$ is the Ω and \mathbf{B} product taken at the region of pair creation.

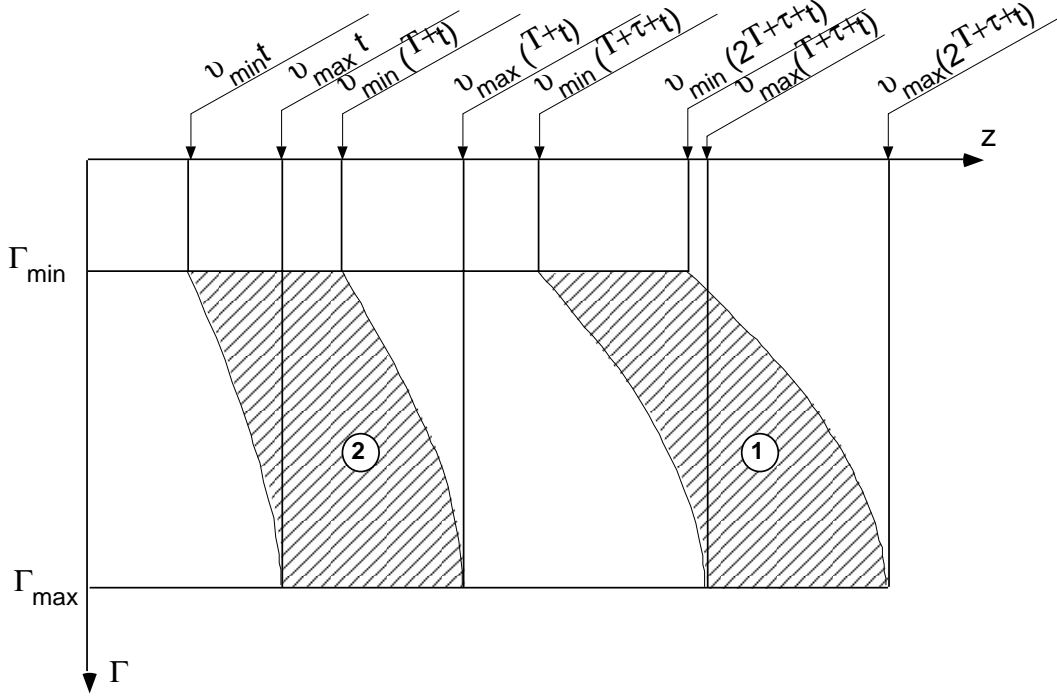


Fig. 4. Particle distribution over the plane with coordinates z and Γ at the instant t . The regions with particles are shaded. The first and the second clouds of plasma are numerated by 1 and 2.

Far the neutron star ($r \gg R$), equations (3) and (11) yield

$$\frac{|\Gamma_+ - \Gamma_-|}{\Gamma_p} \simeq \frac{r\Omega}{c} \frac{\Gamma_p^3}{\Gamma_b} \quad (12)$$

where

$$|\Gamma_+ - \Gamma_-| \simeq \frac{|v_+ - v_-|}{c} \Gamma_p^3. \quad (13)$$

Using a relativistic generalization of the Penrose criteria (Buschauer & Benford 1977; Krall & Trivelpiece 1986) and equation (12), the condition for development of the two-stream instability may be written as

$$\Gamma_p > 2 \times 10^2 (c/r\Omega)^{1/3}. \quad (14)$$

From equation (14) we can see that the two-stream instability, which develops because of the relative motion of the secondary plasma components, may occur only rather near the light cylinder of pulsars. Besides, the magnetic field near the pulsar surface must be close to the dipole field when the Lorentz-factor of the secondary particles is equal to its maximum (see, equation (1)).

4.3. The interaction between clouds of the secondary plasma

Here the nonstationarity of plasma ejection from the polar gap regions is modeled as follows. Plasma particles (electrons and positrons) are ejected along the z -axis from $z = 0$ during the time T with Lorentz-factors ranging from

Γ_{\min} to Γ_{\max} , so that $\Gamma_{\max} \gg \Gamma_{\min} \gg 1$. Then over the time τ no particles are ejected, after that everything is repeated. In other words, we assume that the secondary e^+e^- plasma is ejected in the form of separate isolated clouds with the length cT along the z -axis, and the space between these clouds is $c\tau$.

At $z = 0$, the distribution of ejected particles over Lorentz-factors is described by the function $f_o(\Gamma)$ determined from the condition that the number of particles dn_p per volume dV with energies from $m_e c^2 \Gamma$ to $m_e c^2 (\Gamma + d\Gamma)$ is equal to

$$dn_p = n_p dV f_o(\Gamma) d\Gamma. \quad (15)$$

The distribution function thus determined is normalized to unity, i.e.,

$$\int_{\Gamma_{\min}}^{\Gamma_{\max}} f_o(\Gamma) d\Gamma = 1. \quad (16)$$

The pulsar plasma can be regarded as collisionless. Therefore, the spatial and energy distribution of outflowing particles at $z > 0$ may be found from simple kinematic relations.

To study the dispersion of plasma clouds and their mutual overlapping it is sufficient to consider the outflow of two successively ejected clouds. It is expedient to draw the distribution of particles in the z - and Γ -coordinates (see Fig. 4). In our case the initial time ($t = 0$) is the moment when the ejection of particles of the second cloud is just finished.

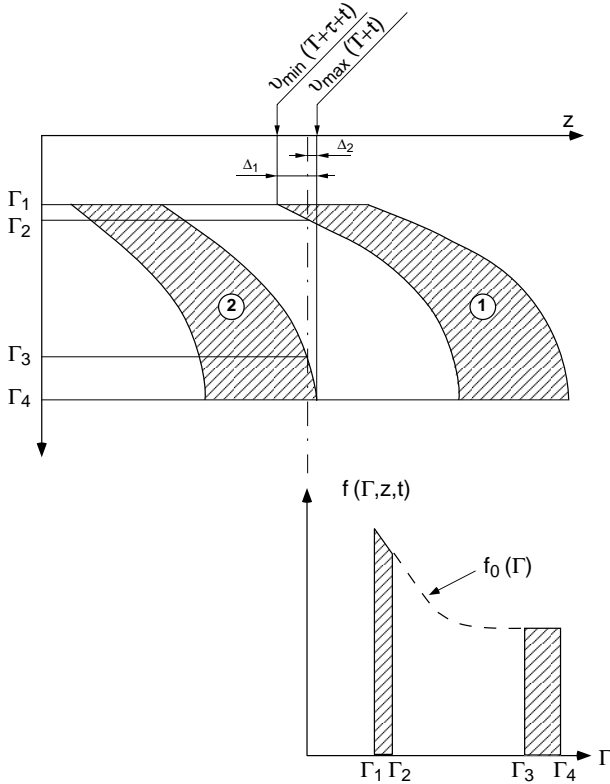


Fig. 5. Overlapping of plasma clouds at $t > t_0$. The regions with particles are shaded. In the overlapping region, z meets the condition $v_{\min}(T + \tau + t) < z < v_{\max}(T + t)$. The distribution function of particles in the overlapping region at the distance z from the pulsar surface is shown in the bottom.

At the time

$$t_o = \frac{v_{\min}(T + \tau) - v_{\max}T}{v_{\max} - v_{\min}} \simeq 2\Gamma_{\min}^2\tau \quad (17)$$

determined from the equation

$$v_{\min}(T + \tau + t_o) = v_{\max}(T + t_o), \quad (18)$$

the fast particles of the second cloud catch up with the slow particles of the preceding cloud, and mutual overlapping of the clouds begins, where

$$v_{\min} = [1 - (1/\Gamma_{\min})^2]^{1/2}c, \quad (19)$$

$$v_{\max} = [1 - (1/\Gamma_{\max})^2]^{1/2}c. \quad (20)$$

The width of the overlapping region at $t > t_o$ is

$$\Delta = v_{\max}(T + t) - v_{\min}(T + \tau + t). \quad (21)$$

The distribution function of particles in the overlapping region is (see Fig. 5)

$$f(\Gamma, z, t) = \begin{cases} 0, & \Gamma < \Gamma_1, \\ f_o(\Gamma), & \Gamma_1 < \Gamma < \Gamma_2, \\ 0, & \Gamma_2 < \Gamma < \Gamma_3, \\ f_o(\Gamma), & \Gamma_3 < \Gamma < \Gamma_4, \\ 0, & \Gamma_4 < \Gamma, \end{cases} \quad (22)$$

where

$$\Gamma_i = [1 - (v_i/c)^2]^{-1/2}, \quad i = 1, 2, 3, 4, \quad (23)$$

$$v_1 = v_{\min}, \quad v_2 = \frac{z}{T + \tau + t}, \quad (24)$$

$$v_3 = \frac{z}{T + t}, \quad v_4 = v_{\max}. \quad (25)$$

In the overlapping region, there are, in fact, two streams of slow and fast particles (see Fig. 5). This may lead to development of the two-stream instability. A simple approximation where plasma is one-dimensional and homogeneous may be employed for consideration of this instability in the overlapping regions of plasma clouds in the magnetospheres of pulsars (Usov & Ursov 1988). In this approximation, the dispersion equation for l -waves with $\mathbf{k} \parallel \mathbf{v} \parallel \mathbf{B}$ is

$$1 - \frac{4\pi e^2 n_p}{m_e} \int \frac{f(\Gamma, z, t) d\Gamma}{\Gamma^3 (\omega - kv)^2} = 0 \quad (26)$$

At the initial stage of clouds overlapping, when the width of the overlapping region is much smaller than the width of the clouds, along the z -axis, $\Delta \ll cT$, the function $f(\Gamma, z, t)$ describes two groups of practically monoenergetic particles at velocities v_2 and v_4 . Taking this into account, equation (26) may be written as

$$1 - \omega_p^2 \left[\frac{\alpha}{(\omega - kv_2)^2} + \frac{\beta}{(\omega - kv_4)^2} \right] = 0 \quad (27)$$

where

$$\alpha = \int_{\Gamma_1}^{\Gamma_2} \frac{f_o(\Gamma)}{\Gamma^3} d\Gamma, \quad \beta = \int_{\Gamma_3}^{\Gamma_4} \frac{f_o(\Gamma)}{\Gamma^3} d\Gamma. \quad (28)$$

From equation (27), the maximum growth rate of l -waves is (Ursov & Usov 1988)

$$\text{Im } \omega_{\max} = \frac{\sqrt{3}}{2} \alpha^{1/2} \left(\frac{\beta}{2\alpha} \right)^{1/3} \omega_p, \quad (29)$$

and occurs for the wave vector

$$k_{\text{res}} = \frac{\alpha^{1/2} \omega_p}{v_4 - v_2} \quad (30)$$

that corresponds to a resonance between plasma oscillations of the slow ($\Gamma_1 < \Gamma < \Gamma_2$) flow and high-energy particles of the fast ($\Gamma_3 < \Gamma < \Gamma_4$) flow.

Since for the slow and fast flows the Γ ranges of plasma particles are narrow for slightly overlapping clouds ($\Delta \ll c\tau$), the function $f_o(\Gamma)$ in equations (28) can be factored out the integral sign and then to integrate:

$$\alpha \simeq \frac{f_o(\Gamma_1)}{2} \frac{\Gamma_2^2 - \Gamma_1^2}{\Gamma_1^2 \Gamma_2^2} = \frac{v_2 - v_1}{c} f_o(\Gamma_1) \simeq \frac{\Delta_1}{ct_o} f_o(\Gamma_{\min}) \quad (31)$$

$$\beta = \frac{f_0(\Gamma_4)}{2} \frac{\Gamma_4^2 - \Gamma_3^2}{\Gamma_3^2 \Gamma_4^2} = \frac{v_2 - v_3}{c} f_0(\Gamma_4) \simeq \frac{\Delta_2}{ct_o} f_0(\Gamma_{\max}), \quad (32)$$

where (see Fig. 5)

$$\Delta_1 = z - v_{\min}(T + \tau + t), \quad \Delta_2 = v_{\max}(T + t) - z \quad (33)$$

in the overlapping region,

$$v_{\min}(T + \tau + t) < z < v_{\max}(T + t). \quad (34)$$

Substituting equations (31) and (32) into equation (29), we obtain

$$\begin{aligned} \text{Im } \omega_{\max} &= \frac{\sqrt{3}}{2^{4/3}} \omega_p [f_o(\Gamma_{\min})]^{1/6} [f_o(\Gamma_{\max})]^{1/3} \\ &\times \left(\frac{\Delta - \Delta_2}{ct_o} \right)^{1/6} \left(\frac{\Delta_2}{ct_o} \right)^{1/3}. \end{aligned} \quad (35)$$

The growth rate (35) depends on the distance Δ_2 to the leading front of the cloud overlapping and reaches the maximum

$$\begin{aligned} (\text{Im } \omega_{\max})_{\max} &= \frac{\omega_p}{2} [f_o(\Gamma_{\min})]^{1/6} [f_o(\Gamma_{\max})]^{1/3} \\ &\times (\Delta/ct_o)^{1/2} \end{aligned} \quad (36)$$

at $\Delta_2 = (2/3)\Delta$.

The distribution function of plasma particles in the magnetospheres of pulsars may be roughly approximated by the following law (Arons 1981)

$$f_o(\Gamma) = \begin{cases} A \Gamma^{-3/2} & \Gamma_{\min} < \Gamma < \Gamma_*, \\ A \Gamma_*^{-3/2} & \Gamma_* < \Gamma < \Gamma_{\max}, \end{cases} \quad (37)$$

where A is the constant determined from the normalization condition (16) and written as

$$A = \frac{\Gamma_{\min}^{1/2}}{2} \left[1 + \frac{1}{2} \left(\frac{\Gamma_{\min}}{\Gamma_*} \right)^{1/2} \left(\frac{\Gamma_{\max}}{\Gamma_*} - 3 \right) \right]^{-1}. \quad (38)$$

Substituting equation (37) into equation (36) and taking into account that $ct_o \simeq 2L\Gamma_{\min}^2$, we have the characteristic time of the instability development

$$\tau_i = (\text{Im } \omega_{\max})_{\max}^{-1} = \frac{2^{3/2} \Gamma_{\min}^{5/4} \Gamma_*^{1/2}}{\omega_p A^{1/2}} \left(\frac{L}{\Delta} \right)^{1/2}, \quad (39)$$

where $L = c\tau$ is the space between the plasma clouds.

An instability manages to develop if the lifetime of the unstable system exceeds the characteristic time of the instability development by more than an order. In our case, the unstable system is the overlapping region, and its lifetime is Δ/c . With a similar criterion, $10\tau_i = \Delta_i/c$,

we get the value of Δ at which the two-stream instability is developing to be given by

$$\Delta_i \simeq (30c\omega_p^{-1} A^{-1/2} \Gamma_{\min}^{5/4} \Gamma_*^{1/2} L^{1/2})^{2/3}. \quad (40)$$

We now estimate the quantities that characterize the development of the two-stream instability in the magnetospheres of pulsars. The following parameters of the secondary e^+e^- plasma may be assumed: $\Gamma_{\min} \simeq 5$, $\Gamma_* \simeq 10^2$, $\Gamma_{\max} \simeq 10^3$ (Arons 1981) and $L = c\tau \simeq cT \simeq R \simeq 10^6$ cm (Usov 1987). From equations (3)-(5), (38) and (40), we have $\Delta_i \simeq 0.8 \times 10^5$ cm. Hence, the development of the two-stream instability already occurs when the cloud overlapping is very small ($\Delta_i \ll L$). Further overlapping of the plasma clouds ($\Delta > \Delta_i$) could be considered only beyond the scheme of our linear approximation.

The distance from the neutron star to the region where the two-stream instability occurs is

$$r_i \simeq 2c\tau\Gamma_{\min}^2 = 2L\Gamma_{\min}^2 \quad (41)$$

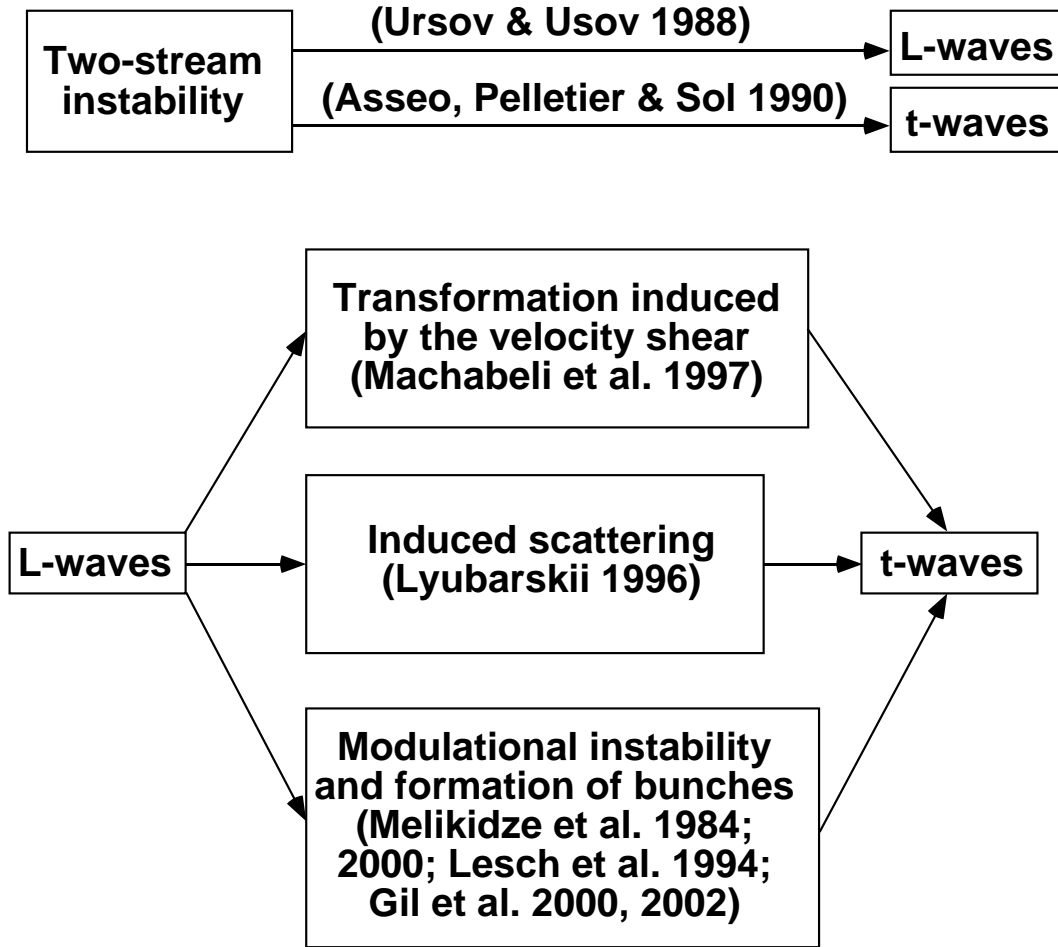
or numerically $r_i \simeq 5 \times 10^7$ cm for $L \times 10^6$ cm and $\Gamma_{\min} \simeq 5$. The value of r_i is consistent with the distance from the neutron stars to the radio emission regions (see equation (9)).

The existence of several scales of the outflowing plasma modulation (e.g., $L_1 \simeq 0.3R$ and $L_2 \simeq 2R_c$ in the Ruderman-Sutherland model) may lead to the situation that in the magnetospheres of pulsars there are several regions where the two-stream instability develops and the generation of coherent radio emission occurs. From the observational data on radio emission of pulsars it follows that such a situation is possible for some pulsars (e.g., Gil 1985).

5. Discussion

The mechanism of coherent radio emission of pulsars is closely connected with the location of the radio emitting regions in the pulsar magnetospheres. If it is confirmed that the distances from the neutron stars to the radio emitting regions are $\ll c/\Omega$ (Gil & Kijak 1993; Rankin 1993; Kramer et al. 1994; Kijak & Gil 1997), i.e., these regions are far inside the light cylinder, then most probably, the two-stream instability developed in the outflowing strongly nonhomogeneous plasma (Usov 1987; Ursov & Usov 1988; Asseo & Melikidze 1998) is responsible for the generation of the pulsar radio emission. It is worth noting that if the process of pair creation near the pulsar surface in strongly nonstationary indeed, the development of this instability in the magnetospheres of pulsars is almost inevitable.

Longitudinal (nonescaping) Langmuir waves are generated in the process of development of the two-stream instability. Then, these waves may be converted into t -waves, that can escape from the pulsar magnetosphere, by means of the following nonlinear effects:



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